

Verifiable Elections with Commitment Consistent Encryption A Primer

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Abstract

This note provides an introduction to the PPATS Commitment Consistent Encryption (CCE) scheme proposed by Cuvelier, Pereira and Peters [7] and its use in the design of end-to-end verifiable elections with a perfectly private audit trail. These elections can be verified using audit data that will never leak any information about the vote, even if all the private keys of the elections are compromised, or if the cryptographic assumptions are broken.

1 Introduction

The verification process of end-to-end verifiable elections includes the examination of a set of audit data made available, usually through a public website, to anyone who would like to verify the election. Once these audit data are released, they can be copied by numerous people around the world and stored for ever. In order to make sure that these data *hide* the votes but still *bind* the votes to the election result, that is, prove that this result is correct, they are typically produced using various cryptographic techniques.

In most end-to-end verifiable voting schemes, these hiding and binding properties of the audit data are not absolute. They rely on the assumption that pieces of secret information held by some trustees remain secret and on the assumption that solving some mathematical problems cannot be done because it would require way too much computational power. This is the case, for instance, when the audit data contain votes that are encrypted with a public key encryption scheme (ElGamal, Paillier, . . .), or are hidden thanks to the use of a pseudorandom generator, or when the Fiat-Shamir heuristic is used to produce efficient non-interactive validity proofs.

The two flavors in which non absolute security properties appear, that is, depending on keeping secrets and depending on computational assumptions, are actually fairly different. Stealing secret information is something that may happen at any time or, hopefully, will not happen at all. But, if it happens, it is

likely to happen during the election, as this is when the secrets are manipulated — they should normally be destroyed after that. By contrast, computational advances are expected to happen no matter what, but we can try to estimate their pace and reasonably hope that they will not happen before several years.

In some cases though, we can design schemes that have absolute or perfect security properties. We know that these schemes will never be broken, independently of any stolen keys or computing advances. Unfortunately, we also know that, under natural settings, we cannot have a voting scheme that offers the perfect flavors of the binding and hiding properties.

Let us now consider the consequences of stolen secrets and computational advances on our hiding and binding properties. Regarding the binding property, all end-to-end verifiable systems are designed in such a way that the correctness of the result does not depend on secret information kept by trustees: these trustees would de facto be able to fake the election results, which is far more trust than we are ready to give them. The impact of computational progresses on the binding property still exists, but is much more benign: if some advances make it possible, in several years, to create new audit data that would pass an election verification procedure and support a different result, such data will not convince anyone since we will know that the strength of the cryptographic schemes on which the verification procedure relies is gone. Furthermore, the election results would have been validated for a long time, and it would be impossible to change anything anyway. So, proofs must be available when they are needed, but may become unconvincing in the future, when they do not matter anymore.

Regarding the hiding property, stolen secret information would typically result in the loss of privacy of the votes, which certainly is very damaging. The eventual compromise of the privacy of the votes due to computational advances is also a problematic threat: this perspective can be sufficient to enable coercion, which is the main threat that the secret ballot is expected to prevent.

The discussion above is summarized in Table 1. This table suggests that targeting a perfect flavor of the hiding property is quite appealing, as it would eliminate the threats from the last line of the table. This imposes keeping a computational flavor of verifiability, but this looks acceptable as long as this verifiability does not rely on keeping secrets. These are the properties we are targeting in this note, which are also summarized in Table 2.

| | compromise of secret information | computational advances in the future |
|------------------------|---|--------------------------------------|
| Binding breaks due to: | Cannot happen in E2E verifiable systems | Benign effect |
| Hiding breaks due to: | Highly Damaging | Damaging and unavoidable |

Table 1: Effect of cryptographic issues on the binding and hiding properties of public audit data in most end-to-end verifiable election systems.

| | compromise of secret information | computational advances in the future |
|------------------------|----------------------------------|--------------------------------------|
| Binding breaks due to: | Cannot happen | Benign effect |
| Hiding breaks due to: | Cannot happen | Cannot happen |

Table 2: Effect of cryptographic issues on the binding and hiding properties of public audit data when using a perfectly private audit trail.

2 Commitment-Consistent Encryption

Commitment-consistent encryption (CCE) schemes [7] are cryptographic mechanisms that can provide verifiable elections with the properties described in Table 2.

In many large scale verifiable election schemes, votes are transmitted to a set of trustees under the protection of a public key encryption mechanism. However, the ciphertexts produced by a public key encryption mechanisms cannot be perfectly hiding and, therefore, cannot be part of our election audit data.

Commitment-consistent encryption schemes are public key encryption schemes with an extra feature that addresses this issue: from any encrypted vote, it is possible to extract a perfectly hiding commitment that is consistent with that vote. That commitment can be part of the audit data, be verified by anyone, and we are sure that it cannot be manipulated without breaking the binding property of the commitment scheme.

2.1 Computational setting

We are looking for efficient instances of CCE schemes that can be easily used in a distributed setting. For this reason, the schemes that we discuss here work with elements of public prime order groups. This in particular enables efficient distributed key generation [9] and the manipulation of integers that are shorter than those occurring in schemes that rely on the hardness of factoring.

So, we are going to compute in cyclic groups of prime order q . We refer to these groups with the letter \mathbb{G} , possibly indexed, and use the letters g and h to denote generators of such groups.

The computational security of our schemes is guaranteed as long as the Decisional Diffie-Hellman (DDH) problem [3] is hard. Informally, this problem consists in deciding whether a tuple is sampled from the distribution (g, g^x, g^y, g^{xy}) or from the distribution (g, g^x, g^y, g^z) for uniformly random $(x, y, z) \leftarrow \mathbb{Z}_q^3$. It is believed to be hard in large prime order subgroups of \mathbb{Z}_p^* and in various groups obtained from elliptic curves for instance. The hardness of this problem implies the hardness of other related problems, including the discrete logarithm (DL) problem, that states that is hard to compute x from inputs (g, g^x) where x is uniformly random in \mathbb{Z}_q .

2.2 Perfectly hiding commitments

One of the most popular example of perfectly hiding commitment scheme was introduced by Chaum et al. [5], and is often called the Pedersen commitment: given two random generators g_1 and g_2 of the group \mathbb{G}_1 ,¹ we can commit on a vote $v \in \mathbb{Z}_q$, by computing the value $c = g_1^r g_2^v$ for a random element $r \leftarrow \mathbb{Z}_q$. An opening of this commitment is simply made of the values r and v (but we later omit this last value, and assume that it can be efficiently computed by exhaustive search, which is certainly the case when v is a single bit). This scheme is perfectly hiding: since g_1^r is a uniformly random group element, the resulting c is also uniformly distributed in \mathbb{G}_1 (this is equivalent to running a one-time pad in \mathbb{G}_1). It is also binding if the DL problem is hard in \mathbb{G}_1 : any two distinct openings (r_1, v_1) and (r_2, v_2) of the same c would immediately lead to computing the discrete logarithm of g_2 in basis g_1 , which would be equal to $\frac{r_2 - r_1}{v_1 - v_2}$.

These commitments are quite appealing: they have a simple expression and are homomorphic: the product of two commitments can be opened with the sum of the committed values and randomnesses. This property is most useful, as it makes it possible to multiply committed votes in order to obtain a commitment on the sum of these votes.

In order to make a CCE scheme based on this commitment scheme, we would then need to encrypt r (and possibly v as well) with an additively homomorphic encryption scheme, so that the trustees would be able to compute an opening of the product of the commitments on all votes. Unfortunately, traditional additively homomorphic encryption schemes have either exponential decryption time, which is not acceptable given the large size of r , or require using groups of composite order, which we want to avoid for the reasons described above.

Recently, another way of opening these commitments was proposed by Abe et al. [1], and can serve our purpose. The proposal is to use a second group \mathbb{G}_2 of same prime order q , with generator h_1 , and to define the opening of a commitment $c = g_1^r g_2^v$ as $a = h_1^r$: computing that last value indeed seems to require the knowledge of r . However, we need a mechanism to be able to verify that a is really an opening of c for a given vote v . To this purpose, \mathbb{G}_1 and \mathbb{G}_2 are chosen to be groups admitting an efficient bilinear map $e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$, where \mathbb{G}_T is called the target group.² The bilinearity here implies that, if $e(g_1, h_1) = g_T$, then $e(g_1^a, h_1^b) = g_T^{ab}$. Thanks to this bilinear map, it is possible to verify that a opens the commitment c for vote v by checking that $e(c/g_2^v, h_1) = e(g_1, a)$.

These commitments certainly remain perfectly hiding, for the same reason as

¹Such generators can be obtained through a mapping from the outputs of a PRG or from the digits of a public constant like the number π .

²Choices for the groups \mathbb{G}_1 and \mathbb{G}_2 are based on elliptic curves admitting a so-called Type 3 (asymmetric) pairing [8], which provide the most freedom and the most efficient implementations in the pairing landscape. It is not difficult to transpose the schemes we propose to the symmetric setting (Type 1 pairing), but the resulting solutions are less efficient and cannot be based on the DDH problem anymore, since DDH is easy to solve in these groups (the DLIN problem is a natural choice there.)

above. The binding property holds if the DDH problem is hard in \mathbb{G}_1 . Suppose indeed that we get a DDH challenge tuple $(g_1, g_1^x, g_1^y, g_1^z)$ and need to decide whether $z = xy$ or not. We can set $g_2 = g_1^x$ and feed an adversary against the commitment scheme with the bases (g_1, g_2, h_1) . This adversary will provide us with two distinct openings (v_1, a_1) and (v_2, a_2) of a commitment of its choice. We can now verify that $z = xy$ if and only if $e(g_1^z, h_1) = e(g_1^y, (a_1/a_2)^{\frac{1}{v_2-v_1}})$.

So, we have a scheme that provides commitments that are as efficient to compute as Pedersen commitments, but with openings that are group elements. The opening verification operation becomes more expensive, though, as the pairing operation is considerably more demanding than an exponentiation in one of the base groups. However, this extra cost remains low at an election scale as, when using homomorphic tallying techniques, we will only need to verify one commitment opening per question, and not per voter. Furthermore, since all computational operations are performed with public bases and secrets in the exponents, this scheme is fully compatible with the most efficient sigma proof techniques.

2.3 The PPATS scheme

In order to have an election with a perfectly private audit trail, we ask voters to commit on their votes using the scheme we just described. Talliers are then expected to publish the election result, and show that they can open the product of the committed votes to this result: this is a simple way to prove the correctness of the outcome based on the binding property of the commitment scheme.

The talliers then need a way to compute that opening, which simply is the product of the openings of the individual votes. To this purpose, the voters also send the opening of their individual votes to the talliers, protected using an additively homomorphic threshold encryption scheme in order to make sure that no subset of the talliers smaller than the chosen threshold would be able to obtain information about individual votes. ElGamal encryption can be used for that purpose and pairs nicely with the commitment scheme, as its security also relies on the hardness of the DDH problem.

Protecting commitment openings with encryption can place the trustees in a difficult position, though: they become unable to determine whether the encrypted information that is sent to them really contains an opening of a vote commitment, or just an arbitrary value. The second case could lead to a very uncomfortable situation, as the trustees would become unable to compute the election results, and would have a hard time proving that they are acting in good faith without violating or reducing the privacy of honest voters. A simple solution to this issue is to require voters to prove the consistency of the CCE ciphertexts they produce. This can be done using a sigma proof, thanks to the structure of our commitment openings.

The resulting scheme is called the PPATS encryption scheme, and is described in Table 3 (for efficiency reasons and consistency with the literature, we

switch the roles of \mathbb{G}_1 and \mathbb{G}_2 compared to our text above: operations in \mathbb{G}_2 are usually more expensive than those in \mathbb{G}_1 .)

The PPATS encryption scheme is additively homomorphic, but its decryption procedure can be quite slow if the values to be decrypted are (very) large, since the extraction of a discrete logarithm is required. This should not be a problem for elections with homomorphic tallying (values up to 2^{50} can still be extracted in seconds using the baby-step giant-step algorithm), but may become an issue if unpredictable write-ins need to be taken into account for instance. For this purpose, the PPATC encryption scheme [7] can be used, which enables efficient decryption and is compatible with efficient mixnets but is not additively homomorphic anymore.

3 Voting based on PPATS encryption

The PPATS scheme can be conveniently used to build a voting scheme with homomorphic tallying. We outline the main steps of a simple voting scheme here. This process can of course be refined with various standard enhancements (cast or audit procedure, ...) or adapted to completely different E2E voting schemes.

1. Groups are chosen, depending on the security parameter, and made public.
2. Trustees generate a PPATS key in a threshold manner. Any protocol (see, e.g., Gennaro et al. [9]) that can be used for DDH-based cryptosystems can be used for PPATS encryption.
3. A public bulletin board is created, with the election description and the PPATS public key (including information that may be necessary to verify the validity of this key.)
4. Voters prepare their ballot by producing a PPATS encryption of a 0 or a 1 for each response to the questions. They also publicly prove the validity of their vote by submitting, for the commitment included in each ciphertext, a proof of knowledge $\sigma_{0/1}$ of an opening of d to a 0 or a 1.³
5. When receiving a ballot, the trustees (or the bulletin board, by delegation) check the validity of all PPATS validity proofs and, if they check, the bulletin board publishes the commitments extracted from these ciphertexts together with the 0/1 validity proofs on these commitments.
6. At the end of the election day, the trustees:
 - (a) check the validity of all proofs posted on the bulletin board. If the proofs check, they

³For commitment d , this can be a disjunctive proof of knowledge of the discrete logarithm of either d or d/g_2 in basis g_1 [6].

PPATS encryption

- *Setup*: Select type-3 pairing-friendly groups $\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T$ of prime order q , together with random generators g_1 of \mathbb{G}_1 and h_1, h_2 of \mathbb{G}_2 .
- *Key Generation*: Generate an ElGamal public encryption key $g_2 = g_1^x$. The secret key is x , possibly existing under a distributed (threshold) form only.
- *Encryption*: Encrypt vote v as $(c_1, c_2, d, \sigma_{cc}) = (g_1^s, g_1^r g_2^s, h_1^r h_2^v, \sigma_{cc})$, using uniformly random $(r, s) \leftarrow \mathbb{Z}_q^2$, and the consistency proof σ_{cc} described in Table 4.
- *Decryption*: Extract the discrete logarithm of $e(c_1^x/c_2, h_1)e(g_1, d)$ in basis $e(g_1, h_2)$. (The computation of c_1^x can be done in a distributed manner.)
- *Extraction of commitment*: The perfectly hiding commitment extracted from a ciphertext $(c_1, c_2, d, \sigma_{cc})$ is d .
- *Extraction of commitment opening*: The commitment opening is computed as $a = c_2/c_1^x$.
- *Opening verification*: Given a commitment d and an opening a for vote v , verify if $e(a, h_1) = e(g_1, d/h_2^v)$.

Table 3: The PPATS encryption scheme

PPATS Consistency Proof for triple $c = (g_1^s, g_1^r g_2^s, h_1^r h_2^v)$

- *Commitment computation*: Compute $c' = (c'_1, c'_2, d') = (g_1^{s'}, g_1^{r'} g_2^{s'}, h_1^{r'} h_2^{v'})$ using uniformly random $(r', s', v') \leftarrow \mathbb{Z}_q^3$
- *Challenge computation*: Compute $e = \mathcal{H}(c, c', label)$. Here, \mathcal{H} is a cryptographic hash function and $label$ is a global public value that contains the description of the groups that are used, together with the generators of \mathbb{G}_1 and \mathbb{G}_2 used in the proof. It should also contain some identifiers for the election and the purpose of the proof.
- *Response computation*: Compute $f_r = r' + er$, $f_s = s' + es$ and $f_v = v' + ev$.

The proof is defined as $\sigma_{cc} = (e, f_r, f_s, f_v)$.

PPATS Validity Verification for triple $c = (c_1, c_2, d)$ and proof $\sigma_{cc} = (e, f_r, f_s, f_v)$.

- *Commitment reconstruction*: Compute $c' = (c'_1, c'_2, d')$ as follows:
 $c'_1 = g_1^{f_s}/c_1^e$, $c'_2 = g_1^{f_r} g_2^{f_s}/c_2^e$, $c'_3 = h_1^{f_r} h_2^{f_v}/d^e$.
- *Challenge verification*: Verify if $e = \mathcal{H}(c, c', label)$.

The validity verification returns the result of the test above.

Table 4: PPATS Validity Proof

- (b) multiply the PPATS ciphertexts from all voters, response by response (removing the consistency proofs), obtaining one ciphertext per response encrypting the election results. Then, they
- (c) extract and publish the commitment opening for each of the resulting ciphertext, and publish the decryption of the election results. Eventually, they
- (d) erase all secret keys.

7. Voters can verify the election as follows:

- (a) Check the public parameters of the election (cryptographic parameters, questions, voter list, ...);
- (b) Check the proper generation of the election public key;
- (c) Check the validity of all votes committed on the bulletin board;
- (d) Multiply the commitments from all voters, response by response, obtaining one commitment per response, committing on the election results;
- (e) Check that the openings provided by the trustees match these commitments.

This process is outlined and described in terms of the PPATS scheme in Figure 1.

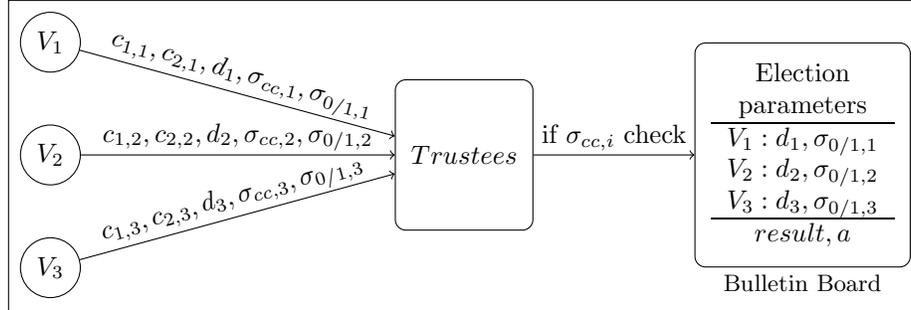


Figure 1: Voting process representation for a single question, based on PPATS. V_1 , V_2 and V_3 are voters. On the bulletin board, $result$ is the decryption of $c = (\prod c_{1,i}, \prod c_{2,i}, \prod d_i)$ and $a = \prod c_{2,i} / \prod c_{1,i}^x$.

This voting scheme satisfies the properties we were looking for. We have E2E verifiability in the traditional sense: the correctness of the outcome does not depend on any trustee, or on any secret information held by anyone. It does depend on computational assumptions, though: fake results could pass the audit procedure if the hash function used in the ZK proofs does not properly emulate a random oracle, or if the DDH problem happens to be easy. These assumptions are well-known and used in numerous other cryptographic schemes. Regarding

the confidentiality of the votes, we see that the content of the bulletin board is perfectly hiding: it only contains an encryption key that is independent of the votes, perfectly hiding commitments and perfect zero-knowledge proofs for all votes and an opening of the public election results. So, these audit data cannot help an adversary that would gain possession of the keys held by the trustees, or would be able to solve computational problems that are believed to be hard today.

4 Security Parameters and Efficiency Notes

Group selection. The most standard choice of curves admitting a type-3 pairing at the 128 bit security level is the BN curves [2], with 256 bit group order. Various implementations of these curves are available, including in the PBC library [10], the MIRACL cryptographic SDK [4], and the RELIC toolkit [11].

To provide a rough idea of running times, the benchmarks of the MIRACL SDK indicate for these curves that exponentiations take .22 ms in \mathbb{G}_1 , .44 ms in \mathbb{G}_2 , and that the pairing operation takes 20 ms (all on a 2010 Intel i5 520M processor). If we focus on the cost of exponentiations when computing a ballot (which is the dominant factor), we can count that a PPATS encryption of a choice together with a 0/1 proof for the commitment takes 6 exponentiations in \mathbb{G}_1 and 5 exponentiations in \mathbb{G}_2 , for a total computing time of 3.52 ms. So, a modern processor should be able to encrypt around 280 responses per second using a single thread, without any precomputation.

Precomputation. Most of the computational work can be performed out of critical moments, which can be useful in elections with large ballots. For instance:

- All the exponentiations needed for computing a PPATS ciphertext are independent of the actual choices made by the voter. A voting client can then perform these operations in advance, while the voter makes his selections for instance.
- All the exponentiations that are needed for ballot preparation and proof verification are in fixed public bases, enabling the efficient use of various precomputation methods, that usually provide a speedup factor between 2 and 3 for usual parameters.
- The validity of all proofs can be checked during the election day: there is no need to wait until the closing of the polls.
- The PPATS ciphertexts can be multiplied together as they come. In this way, the encrypted results can be available instantly when the polls close.
- The discrete logarithm extraction that is part of the decryption of the election results can be immediate if all the powers of the DL basis have been precomputed and stored, which is easy for any realistic election size.

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